$$\sigma'_\theta = \frac{\sigma_\theta - \sigma_r}{2} = \tau \qquad (3)$$

$$\sigma'_r = -\frac{\sigma_\theta - \sigma_r}{2} = -\tau$$

Thus, the equivalent shear stress in a thick-walled cylinder is given by $\tau = (\sigma_\theta - \sigma_r)/2$.

To analyze the unsteady creep behavior of a thick-walled cylinder, three independent conditions must be satisfied throughout the cylinder wall.

(1) Equilibrium of Forces

Radial equilibrium as defined by Lamé's equation:

$$\sigma_\theta - \sigma_r = r\,\frac{d\sigma_r}{dr} \qquad (4)$$

Axial equilibrium:

$$\int_a^b 2\pi r\sigma_z dr = \pi p a^2$$

However, if zero axial creep is assumed, it is easy to show that if Lamé's equation is satisfied then axial equilibrium is automatically satisfied.

(2) Compatibility of strains

$$\epsilon_r - \epsilon_\theta = r\,\frac{d\epsilon_\theta}{dr} \qquad (5)$$

(3) Stress-Strain-Time Relationship for Material

THE BAILEY THEORY

In 1951, Bailey [2] produced a theory for the primary creep behavior of thick-walled cylinders under internal pressure. In this he neglected elastic strains and assumed zero axial creep; he also ignored wall thinning. Consequently, the Bailey theory predicts a stress distribution that is constant with respect to time. The expressions used for the creep strain rates in the tangential, radial, and axial directions are of the following form:

$$\dot\epsilon_\theta = F(J_2)\,\sigma'_\theta\, t^{m-1}$$

$$\dot\epsilon_r = F(J_2)\,\sigma'_r\, t^{m-1} \qquad (6)$$

$$\dot\epsilon_z = F(J_2)\,\sigma'_z\, t^{m-1} = 0$$

where $F(J_2)$ is a simple power function of the second stress invariant J_2.

If, however, torsion creep data is used that can be represented by an expression of the form

$$\gamma = B\tau^n t^m \qquad (7)$$

then Eqs. (6) are not necessary and the Bailey theory becomes much simpler, with the conditions to be satisfied being

$$\sigma_\theta - \sigma_r = 2\tau = r\,\frac{d\sigma_r}{dr}$$

$$\epsilon_\theta - \epsilon_r = \gamma = -r\,\frac{d\gamma}{dr} \qquad (8)$$

$$\gamma = B\tau^n t^m$$

Combination of Eqs. (8) leads to the following expression for the equivalent shear stress at any radius r, viz.

$$\tau = \frac{p}{n(K^{2/n} - 1)}\,\frac{(b)^{2/n}}{(r)} \qquad (9)$$

and the shear strain

$$\gamma = B\left\{\frac{p}{n(K^{2/n}-1)}\right\}^n \frac{(b)^2}{(r)}\, t^m$$

from which the circumferential strain at the outside diameter is

$$\epsilon_{\theta_b} = \gamma_{b/2} = \frac{B}{2}\left[\frac{p}{n(K^{2/n}-1)}\right]^n t^m \qquad (10)$$

THEORY OF JOHNSON, HENDERSON, AND KHAN

This theory [3] takes into account the effect of elastic strains in the creep of a thick-walled cylinder, and the general expression for the strains is of the form

$$\dot\epsilon_\theta = F(J_2)\,\sigma'_\theta\, t^{m-1} + \frac{1}{E}\,\frac{d}{dt}\,\sigma_\theta - \mu(\sigma_r + \sigma_z) \qquad (11)$$

Use of these expressions gives a stress system that is changing with time. However, Larke and Parker [4] have shown that even with this basic difference the strains at the outside surface and the bore as predicted by Bailey and Johnson et al. are identical. Hence, if one is interested purely in strains, there is no need to get involved with the much more complex mathematical theory of Johnson et al.